

Transition to finger convection in double-diffusive convectionM. Kellner¹ and A. Tilgner¹*Institute of Geophysics, University of Göttingen, Friedrich-Hund-Platz 1,
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Finger convection is observed experimentally in an electrodeposition cell in which a destabilizing gradient of copper ions is maintained against a stabilizing temperature gradient. This double-diffusive system shows finger convection even if the total density stratification is unstable. Finger convection is replaced by an ordinary convection roll if convection is fast enough to prevent sufficient heat diffusion between neighboring fingers, or if the thermal buoyancy force is less than 1/30 of the compositional buoyancy force. At the transition, the ion transport is larger than without an opposing temperature gradient.

I. INTRODUCTION

If convection occurs in natural systems, the density of the convecting fluid is frequently affected by several quantities. The most widely studied example is convection in ocean water, where both temperature and salinity determine the density. Different flow structures may occur in convecting ocean water as opposed to distilled water because heat and salinity have different diffusivities¹⁻⁴. Salt fingers are the best known of these double-diffusive structures. Fingers are narrow vertical columns in which fluid is moving vertically. They require a stabilizing temperature and a destabilizing salt concentration gradient. They allow convective transport even if the total density stratification is stable. Fingers form because heat diffuses more rapidly than ions. The fingers must be narrow enough so that heat diffuses between neighboring fingers, but broad enough so that salinity cannot be significantly exchanged between one finger and the next⁵⁻⁷.

According to the commonly accepted picture for finger formation, narrow fingers should not appear if the stabilizing temperature gradient is weak enough so that density increases with height due to the destabilizing salinity gradient. In this case, a convection roll of the same form as observed in ordinary Rayleigh-Bénard convection is expected. These convection rolls have in order of magnitude the same width and height and therefore suffer less from dissipative losses as long and narrow fingers. One naively expects convection rolls to supercede fingers as long as the fluid is top heavy. It thus came as a surprise that a recent experiment found finger convection even in unstably stratified fluids with a weak stabilizing temperature gradient⁸. The present paper explores more thoroughly the conditions for the existence of fingers. It is shown that fingers in the top heavy fluid are really a form of convection distinct from a convection roll and that convection undergoes a genuine transition as control parameters are varied and the system switches from fingers to rolls. This removes doubts that fingers are only a metastable form of convection which disappears if the system is given enough time. The new measurements reported here locate the transition in parameter space and allow us to test various hypotheses concerning the necessary conditions for finger formation.

The next section summarizes the experimental procedures. Apart from the use of thermochromic liquid crystals and minor improvements in temperature control, the apparatus employed here is exactly the same as in Ref. 8. Full details of the system are given there

and will only be repeated in the next section to the extent necessary to make the paper self contained. The third section presents the results.

II. THE EXPERIMENT

The experiments maintain thermal and ion concentration gradients across a convecting fluid by an electrochemical technique^{8,9}. A cell made of copper top and bottom plates and plexiglass side walls is filled with a dilute solution of $CuSO_4$ in sulfuric acid. The copper plates serve both as temperature controlled boundaries and as electrodes. When a potential difference is applied between the two electrodes, a current flows through the cell with copper ions dissolving from one electrode and reattaching to the other. The sulfuric acid does not participate in the electrochemical reaction, but its free ions screen the electric field from the bulk of the cell. Apart from microscopic boundary layers, the copper ions diffuse and are advected through the cell, but do not experience any electric field. The temperature and the copper ion concentration are the two agents determining the density of the fluid.

The material properties entering the problem of double-diffusive convection are the kinematic viscosity of the fluid ν , the diffusivities of temperature and ion concentration, κ and D , and two expansion coefficients α and β determining variations of density ρ as a function of temperature T and copper ion concentration c around a reference state with density, temperature, concentration and pressure ρ_0 , T_0 , c_0 and p_0 via

$$\alpha = -\frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial T} \right)_{c_0, \rho_0, p_0}, \quad \beta = \frac{1}{\rho_0} \left(\frac{\partial \rho}{\partial c} \right)_{T_0, \rho_0, p_0}. \quad (1)$$

Both α and β are positive. Additional control parameters are the gravitational acceleration g , the cell height L , and the temperature and concentration differences applied across the cell, ΔT and Δc , defined as

$$\Delta T = T_{\text{bottom}} - T_{\text{top}}, \quad \Delta c = c_{\text{top}} - c_{\text{bottom}} \quad (2)$$

with the subscripts indicating the boundary at which temperature T or concentration c are evaluated. The concentration difference Δc is only known if the cell is operated at the limiting current^{8,9}. In that situation, $\Delta c = 2c_0$, where c_0 is the average concentration of copper in the solution.

Four non-dimensional numbers are necessary to specify all control parameters of double-diffusive convection. We will use the Prandtl and Schmidt numbers, Pr and Sc , defined

as

$$Pr = \frac{\nu}{\kappa} \quad , \quad Sc = \frac{\nu}{D}. \quad (3)$$

These are material properties which we will consider to be constants for the purpose of this paper with $Pr \approx 8.7$ and $Sc \approx 1970$. These average numbers are slightly different from those given in ref. 8 because the measurements presented here focus on a transition and sample only a subspace of the accessible parameter space. A summary of the data obtained near the transition is given in table I. For all the measurements listed in this table, Pr varies between 8.55 and 8.87, and Sc varies between 1910 and 2050.

The driving forces are characterized by the thermal and chemical Rayleigh numbers, Ra_T and Ra_c , given by

$$Ra_T = \frac{g\alpha\Delta TL^3}{\kappa\nu} \quad , \quad Ra_c = \frac{g\beta\Delta cL^3}{D\nu}. \quad (4)$$

With the sign conventions introduced above, a negative Rayleigh number indicates a stable stratification, which means that in the finger regime, Ra_T is negative and Ra_c positive. Another quantity of interest is the density ratio Λ which quantifies the ratio of thermal and chemical buoyancy and is given by

$$\Lambda = \frac{Ra_T}{Ra_c} \frac{\kappa}{D} = \frac{\alpha\Delta T}{\beta\Delta c}. \quad (5)$$

Among the observables is the Sherwood number, which is directly proportional to the number of ions transported from top to bottom divided by the purely diffusive current, so that the Sherwood number can be determined by

$$Sh = \frac{jL}{zFD\Delta c} \quad (6)$$

if j is the current density, z the valence of the ion ($z = 2$ for Cu^{2+}) and F Faraday's constant. The Sherwood number plays the same role for the ion transport as the Nusselt number for the heat transport. Measurements of the Nusselt number have not been attempted, as they would require a more accurate temperature monitoring than was implemented in the apparatus.

The velocity field was characterized by PIV. A vertical plane near the middle of the cell was illuminated with a pulsed laser and observed at right angles with a camera. Based on the horizontal and vertical components of velocity in the plane of illumination, three different Reynolds numbers, Re_x , Re_y , and Re will be useful:

$$Re_x = \frac{L}{\nu} \left(\frac{1}{A} \int v_x^2 dA \right)^{1/2} \quad , \quad Re_y = \frac{L}{\nu} \left(\frac{1}{A} \int v_y^2 dA \right)^{1/2} \quad , \quad Re = \sqrt{Re_x^2 + Re_y^2}. \quad (7)$$

The integrals extend over the surface A pictured during the PIV measurements, which typically reaches from plate to plate in the vertical and covers more than half of the cell width in the horizontal. The PIV measurements, together with shadowgraph pictures, allow the measurement of the finger thickness d .

For convenient reference, we repeat here the scaling laws given in Ref. 8 for finger convection:

$$\frac{d}{L} = 0.95 |Ra_T|^{-1/3} Ra_c^{1/9} \quad (8)$$

$$Re = 10^{-6} |Ra_T|^{-1/2} Ra_c \quad (9)$$

$$Sh = 0.016 |Ra_T|^{-1/12} Ra_c^{4/9} \quad (10)$$

In the course of the new experiments, some of the measurements reported in table 1 of Ref. 8 were reproduced. However, the entry with $L = 2cm$, $\Delta T/L = -0.1K/cm$ could not be reproduced, presumably because of insufficient temperature control in the earlier experiment. At those parameters, no fingers were detected in the new experiments.

Thermochromic liquid crystals have been used in a few of the new experiments in order to obtain some information about the temperature field. For those experiments, encapsulated liquid crystal particles of diameters around $50\mu m$ were suspended in the electrolyte and a vertical plane was illuminated with white light and observed at right angles with a CCD camera. The pictures taken by the camera were encoded at each pixel in terms of the three variables hue, saturation and intensity. The hue carries the information about the temperature¹⁰. The dependence of hue on temperature is conveniently determined by taking a picture of the cell with the uniform temperature gradient established before the voltage is applied to the cell. The temperatures of the plates were adjusted to exploit as well as possible the color play of the liquid crystal.

III. RESULTS AND DISCUSSION

It appears from the scaling of the finger width (8) that fingers broaden with decreasing stabilizing $|Ra_T|$, and that they possibly continuously transform into convection rolls, so that there is no transition between two fluid dynamic states, the fingers and the convection rolls. However, this is not true and there is a genuine transition from the finger regime to simple convection. It is convenient to demonstrate the existence of a transition with the

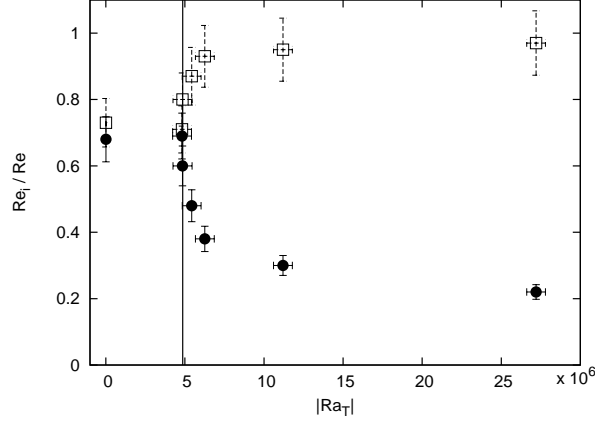


FIG. 1. Re_x/Re (filled circles) and Re_y/Re (empty squares) as a function of $|Ra_T|$ for $Ra_c = 2.92 \times 10^{10}$. The vertical line marks the transitional $|Ra_T|$ deduced from this figure which reappears as one data point in fig. 3.

help of the Reynolds numbers Re_x and Re_y based on the horizontal and vertical velocity components alone. Fig. 1 shows Re_x/Re and Re_y/Re as a function of $|Ra_T|$ at fixed Ra_c . Fingers exist for large $|Ra_T|$. The velocity field is then very anisotropic with a vertical velocity much larger than the horizontal velocity almost everywhere. For small $|Ra_T|$, the fingers disappear and are replaced by a convection roll with an approximately isotropic velocity field with $Re_x/Re \approx Re_y/Re \approx 1/\sqrt{2}$. Fig. 2 provides a visual impression of the various flows encountered during the transition. As fig. 1 shows, the transition from one state to the other occurs at a well defined $|Ra_T|$.

These measurements have been obtained by setting Ra_T to a fixed value and letting the temperature gradient establish itself by diffusion before the voltage was applied to the cell. A few experimental runs were made to exclude the possibility of a hysteresis. In these experiments, the cell was first left to equilibrate in a convection state at a $|Ra_T|$ below the transition to the finger state, before $|Ra_T|$ was suddenly increased to a value above the transition. It was also checked for the opposite direction that no hysteresis appears. Fingers always reappeared with the same amplitude and characteristics as before. In summary, the transition from a convection roll to finger convection is well described as a supercritical bifurcation.

Measurements as shown in fig. 1 have been repeated at six more chemical Rayleigh numbers. The transition line in the $|Ra_T|, Ra_c$ -plane is shown in fig. 3. The measurements

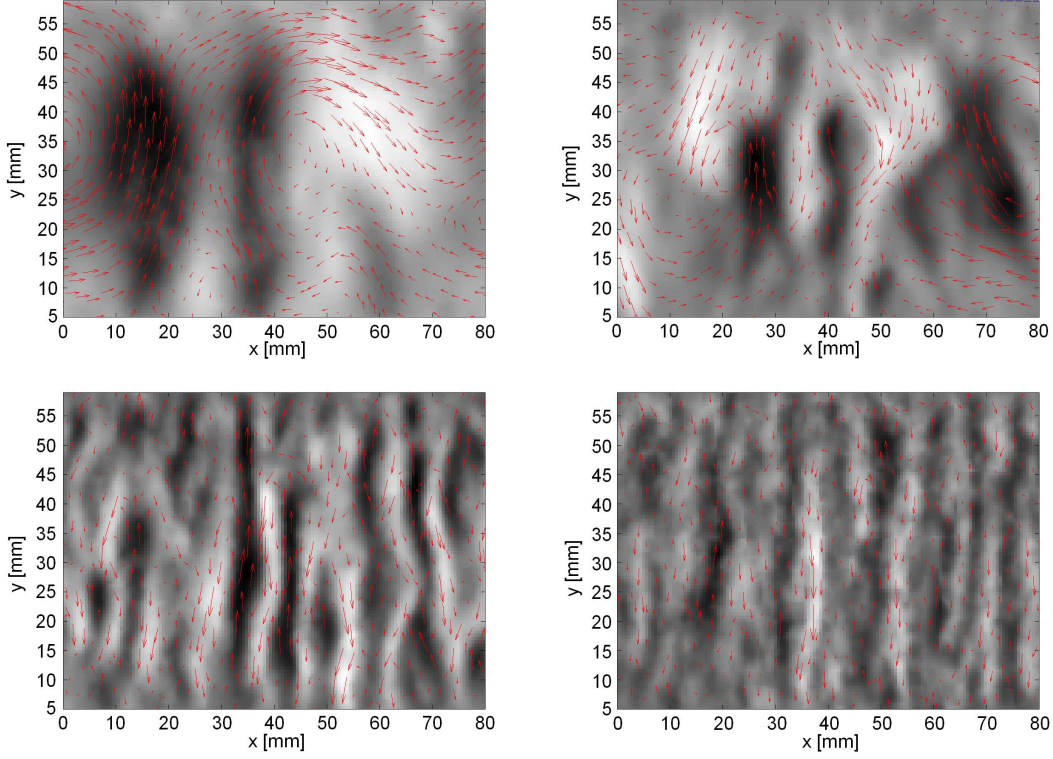


FIG. 2. (Color online) Transition for $Ra_c = 2.92 \cdot 10^{10}$. Shown is the vertical component of the velocity field at $|Ra_T| = 5.94 \cdot 10^3$ (top left), $|Ra_T| = 4.85 \cdot 10^6$ (top right), $|Ra_T| = 1.12 \cdot 10^7$ (bottom left) and $|Ra_T| = 2.72 \cdot 10^7$ (bottom right). The (red) arrows indicate the velocity vector, and the grayshade depends on the vertical velocity component (dark gray in upflows and light gray in downflows). All panels show the velocity field in a vertical plane near the center of the experimental cell. The top and bottom plates are located at $y = 0$ mm and $y = 60$ mm, respectively, and the total lateral extent of the cell is 200 mm.

are compatible with a transition line defined by $Ra_T/Ra_c = \text{const.}$, or $\Lambda = -1/30$. The best approximation to the transition line is a power law given by $|Ra_T| = 8.6 \times 10^{-5} Ra_c^{1.02}$.

It is of course plausible that fingers appear once the stabilizing temperature gradient is strong enough and the density ratio exceeds some threshold value. It is however very surprising that a stabilizing thermal buoyancy as small as $1/30$ of the destabilizing chemical buoyancy ought to be enough to replace the convection roll with fingers. We are therefore led to search for alternative explanations and mechanisms which may govern the transition.

We will start with a few criteria and hypotheses gleaned from previous publications. Ref. 11 has shown that in an infinitely extended medium in which fingers are equivalent to

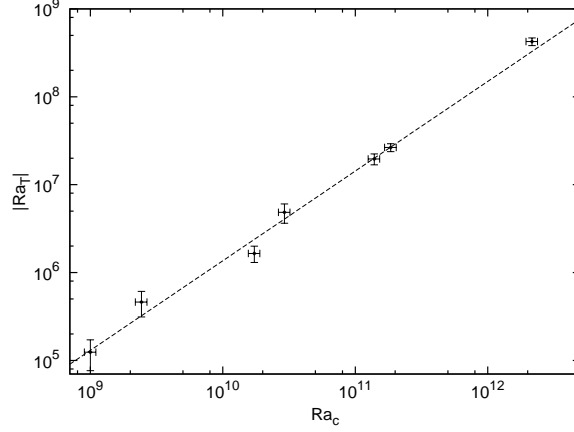


FIG. 3. The transition line between finger convection and convection rolls in the $|Ra_T|, Ra_c$ -plane. The symbols indicate measurements, and the dotted line is a fit given by $|Ra_T| = 8.6 \times 10^{-5} Ra_c^{1.02}$.

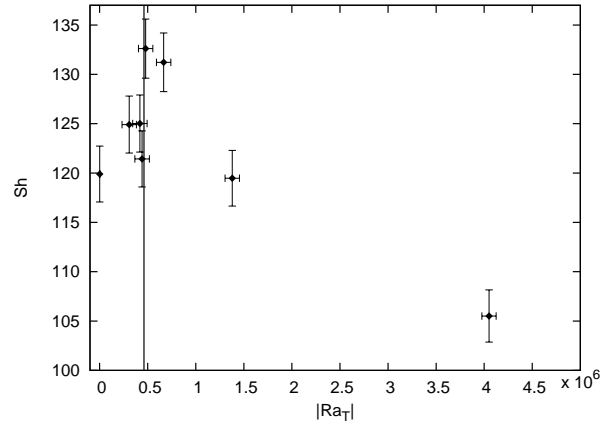


FIG. 4. Sh as a function of $|Ra_T|$ for $Ra_c = 2.43 \times 10^9$. The vertical line marks the transition determined from a measurement as shown in fig. 1.

elevator modes, the fastest growing instability of the linearly stratified ground states is of the finger type for $|\Lambda| > 0.9$ at the Sc and Pr of the experiment. This includes cases with $|\Lambda| < 1$, but does not explain why fingers are observed in the experiment for $|\Lambda|$ as small as $1/30$.

Ref. 12 and ref. 13 showed that fingers become unstable to long internal waves if $Re_f \geq \sqrt{\frac{2Ra_T d^4}{3PrL^4}}$. This relation is obtained from equations (2.1), (2.2), (2.3) and (4.4) of ref. 12 and equation (1.1) of ref. 13 using our notations. In the experiment, the data never satisfy this relation (see table I). We will thus disregard this criterion. Holyer¹⁴ extended

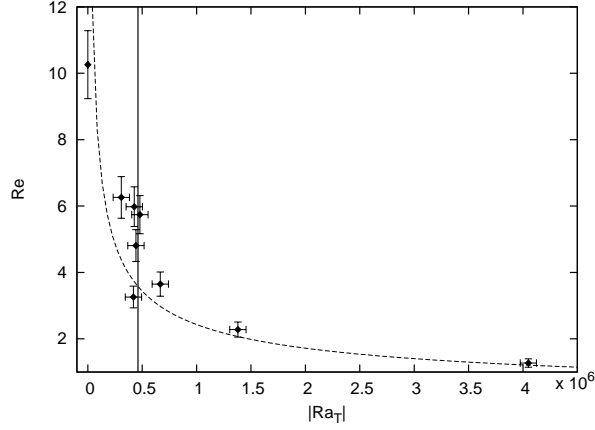


FIG. 5. Re as a function of $|Ra_T|$ for $Ra_c = 2.43 \times 10^9$. The vertical line marks the transition and the dotted line is given by the relation (9).

the stability analysis to include short wavelength disturbances. These disturbances can lead to faster growth rates of the instability than long wavelength modes. However, this work did not result in an analytical criterion and it applies to a basic state in the classical finger regime in which the total density stratification is stable, so that this work is not immediately useful here.

It was already noted in Ref. 8 that the Sherwood number goes to infinity according to eq. (10) if Ra_T tends to zero. Since the temperature gradient is stabilizing in the present configuration, it was speculated that fingers disappear and eq. (10) loses validity at the Ra_T at which the predicted Sh exceeds the value Sh takes in the absence of a stabilizing temperature gradient. This speculation is disproved by fig. 4. Despite the adverse temperature gradient, finger convection can transport more ions than a convection roll in an isothermal fluid. The maximum of Sh is reached at the transition. Fig. 5 demonstrates that the velocity amplitude follows a more intuitive behavior: The smaller $|Ra_T|$, the larger Re . This is of course expected because narrow fingers (which exist at large $|Ra_T|$) increase friction, and because the stabilizing gradient generally opposes motion. The reason why Sh can nonetheless have a maximum is the structure of the velocity field: In a convection roll, vertical velocities which can transport ions from one electrode to the other are only found at the perimeter of the roll, whereas in fingers, up- and downward transport occurs almost everywhere. The most efficient transport thus occurs in the fastest fingers, i.e. at the transition between fingers and convection rolls.

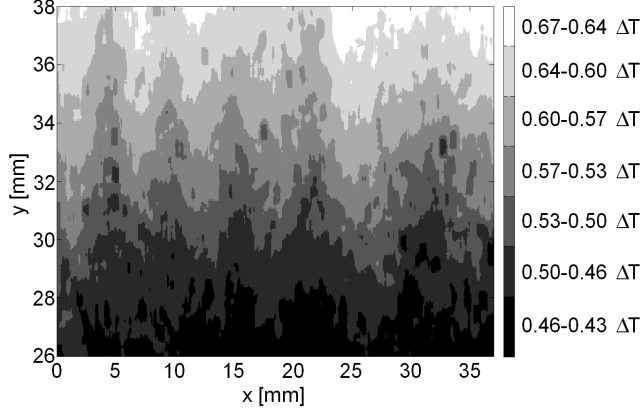


FIG. 6. Gray scale plot of the temperature field visualized in a vertical plane with thermochromic liquid crystals for $Ra_c = 2.92 \cdot 10^{10}$ and $|Ra_T| = 1.86 \cdot 10^7$. Temperatures are grouped into six bins for clarity. The top and bottom plates are located at $y = 0$ mm and $y = 60$ mm, respectively.

Let us next check the hydrodynamic stability of the finger regime. The Reynolds number of individual fingers, based on vertical velocity and finger thickness, is below 1 at the transition, with one exception where it is 1.32 (see table I). It seems excluded that hydrodynamic instability within individual fingers destroys the fingers. The velocity field of the whole collection of fingers is reasonably approximated by what is sometimes called the Kolmogorov flow, defined as $\mathbf{v} = v_{y0} \sin(\pi x/d) \hat{\mathbf{y}}$, where $\hat{\mathbf{y}}$ is the unit vector in y -direction. This flow is stable¹⁵ if $v_{y0}d/\nu < \pi\sqrt{2}$, or $Re_y < \pi$ (since Re_y is defined with the rms rather than the maximum value of velocity). According to table I, this criterion is always met at the transition. Finally, the Rayleigh number of the chemical boundary layer (which has the thickness $L/(2Sh)$ and across which there is a drop in concentration of $\Delta c/2$) is given by $Ra_c/(16Sh)^3$ and is also listed in table I. This Rayleigh number is always less than 250 at the transition. In summary, there is no obvious way to mechanically destabilize the fingers.

We now turn to the temperature field. Fingers only occur within a stable temperature stratification, and one may hypothesize that as $|Ra_T|$ is decreased, the vertical velocity within the fingers increases and destroys more and more the temperature gradient until there is no reason left to form fingers. In order to test this hypothesis, the temperature field was visualized experimentally with thermochromic liquid crystals. An example of a temperature field obtained by this technique is shown in fig. 6. For slow convective motion, one expects the isotherms to be distorted by advection in a sinusoidal fashion as seen in this figure. Since the top and bottom boundaries remain at constant temperature, such a deformation implies that

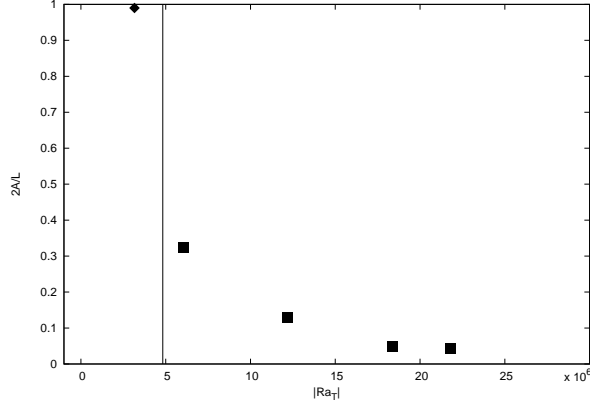


FIG. 7. The peak to peak amplitude $2A$ of the distortion of the isotherm with temperature T given by $(T_{\text{bottom}} - T)/\Delta T = 0.51$, as a function of $|Ra_T|$ for $Ra_c = 2.92 \times 10^{10}$, determined from experimental visualization (filled squares). The diamond denotes a point obtained from the experiment without a detectable temperature gradient. Therefore this value was arbitrarily set to $2A/L = 1$.

the temperature gradient is increased near the top plate in the updrafts and near the bottom plate in the downflows. The advection of the temperature field therefore does not globally reduce the stable stratification as long as the amplitude of the distortion of the isotherms is less than the cell height. In a well mixed state, the temperature outside the boundary layers adjacent to the top and bottom plates is everywhere equal to the arithmetic mean of the temperatures of the top and bottom plates, so that the amplitude of the deformation of the isotherm with the temperature T defined by $(T_{\text{bottom}} - T)/\Delta T = 0.5$ must be larger than half the cell height. The difference $2A$ between the maximum and minimum height of the isotherm with $(T_{\text{bottom}} - T)/\Delta T = 0.51$ was extracted from pictures like fig. 6 for different Ra_T at $Ra_c = 2.92 \times 10^{10}$ and is shown in fig. 7. As the transition is approached from within the finger regime by reducing $|Ra_T|$, the distortion of the isotherms increases, but at the transition, there is still a significant temperature gradient left, whereas there is no detectable temperature stratification below the transition in the convection roll. This is indicated in fig. 7 by an experimental point which we assign the value of $2A/L = 1$. There is therefore no evidence that a reduction of the stabilizing gradient leads to the disappearance of the fingers, since when the transition occurs, there is still a gradient present which in parts of the cell is even larger than the imposed gradient.

Apart from setting up a stabilizing gradient, temperature has another criterion to fulfill for fingers to form: Heat needs to diffuse fast enough. Fingers form because heat is exchanged between neighboring fingers, but ions are not. Heat therefore has to diffuse over a distance larger than the finger thickness during the transit time from one electrode to the other. The ratio of the diffusion length to the finger thickness is given by $\frac{1}{d}\sqrt{\kappa L/V}$ and is also listed in table I. Fingers should disappear if $\frac{1}{d}\sqrt{\kappa L/V} \approx 1$. Inserting the scalings for d and V from eqs. (8) and (9), this predicts the transition to occur on a line in the $|Ra_T|, Ra_c$ -plane defined by $|Ra_T| = 6.6 \times 10^{-6} Pr^{6/7} Ra_c^{22/21}$, where the prefactor results from the requirement that $\frac{1}{d}\sqrt{\kappa L/V}$ be exactly 1. The exponent 22/21 is indistinguishable from the one found in fig. 3 because 22/21=1.05. With this exponent, the best fitting prefactor deduced from the data in fig. 3 is 6.9×10^{-6} .

While the exponent of Ra_c is nearly the same as the one obtained from the transition criterion $\Lambda = const.$, the two criteria have different dependencies on Pr and Sc , but these parameters cannot be varied significantly in the present set-up. The ratio of heat diffusion length and finger thickness varies by a factor of two along the transition line according to table I, but these variations are not systematic and are attributable to uncertainties in the measurements.

We therefore arrive at the picture that fingers exist as long as the diffusion of heat reaches across fingers. As the transitional $|Ra_T|$ is approached from above, velocities increase, transit times decrease and fingers thicken, until a point is reached where the thermal diffusion length during the transit time equals the finger width and fingers disappear. This leaves the question why fingers appear in a top heavy fluid once a convection roll has been established and $|Ra_T|$ is increased beyond the transition at constant Ra_c . We are thus reminded of the fact that we lack a definitive understanding for why convection rolls form in high Rayleigh number convection. One could equally well imagine convection in the form of many independent plumes crossing the cell much as bubbles in a boiling pot of water without a large scale circulation. We know that high Rayleigh number convection does result in large scale convection cells, but as the present experiments show, a small disturbance of 1/30 th of the buoyancy force is enough to disrupt the convection roll and to replace it with a small scale convection structure.

Ra_c	$ Ra_T $	Λ	Re	Re_f	$Ra_{c,\lambda}$	$\sqrt{\kappa L/(vd^2)}$	$\sqrt{\frac{2}{3} \frac{Ra_T}{Pr}} \left(\frac{d}{L}\right)^4$
$1.00 \cdot 10^9$	$1.33 \cdot 10^5$	0.030	03.6	0.59	138	1.45	1.87
	$1.15 \cdot 10^5$	0.026	04.1		140		
$2.43 \cdot 10^9$	$4.79 \cdot 10^5$	0.044	05.7	0.73	065	1.03	4.34
	$4.42 \cdot 10^5$	0.041	04.8		085		
$1.73 \cdot 10^{10}$	$1.78 \cdot 10^6$	0.024	04.2	0.39	112	1.66	4.52
	$1.56 \cdot 10^6$	0.021	04.7		112		
$2.92 \cdot 10^{10}$	$4.85 \cdot 10^6$	0.037	18.9	1.32	110	1.01	4.77
	$4.82 \cdot 10^6$	0.037	34.9		119		
$1.39 \cdot 10^{11}$	$2.05 \cdot 10^7$	0.034	10.8	0.63	192	1.69	5.15
	$1.96 \cdot 10^7$	0.032	09.7		191		
$1.85 \cdot 10^{11}$	$2.83 \cdot 10^7$	0.035	08.8	0.60	159	1.66	7.58
	$2.47 \cdot 10^7$	0.031	14.5		181		
$2.16 \cdot 10^{12}$	$4.51 \cdot 10^8$	0.048	25.2	0.62	240	2.28	5.18
	$3.98 \cdot 10^8$	0.042	29.9		245		

TABLE I. Parameters for the cell adjusted to be near, both above and below, the transition between finger convection and convection rolls: Ra_c , Ra_T , Λ , and Re as defined in the text, together with the Reynolds number based on finger thickness, $Re_f = Re_y d/L$, chemical boundary Rayleigh number $Ra_{c,\lambda} = Ra_c/(16Sh^3)$, the ratio of heat diffusion length to finger thickness, $\sqrt{\kappa L/(vd^2)}$ and the Stern criterion (see Refs. 12 and 13). No value is given for Re_f and in the last two columns for points below the transition where no fingers exist.

IV. CONCLUSION

The transition from finger convection to convection rolls in double-diffusive convection was studied in an electrodeposition cell for $Pr \approx 8.7$ and $Sc \approx 1970$ with stabilizing temperature and destabilizing concentration gradients. Fingers are observed for density ratios $|\Lambda| = \alpha|\Delta T|/(\beta\Delta c) < 1$, i.e. when the system is denser at the top than at the bottom. Fingers are replaced by convection rolls only for $|\Lambda| < 1/30$. At the transition, the ion transport is larger than without an adverse temperature gradient. From various experimental observations, the most plausible mechanism limiting the existence of fingers is heat

diffusion. If the stabilizing temperature gradient is too small, heat has insufficient time to diffuse between neighboring fingers during the transit from one boundary to the other, so that the difference between the diffusivities of heat and ions cannot have an effect any more. This happens for $|Ra_T| \propto Pr^{6/7} Ra_c^{22/21}$, which cannot be distinguished experimentally from $|\Lambda| = 1/30$ because the exponent $22/21$ is too close to 1, and the dependencies on Pr and Sc cannot be tested. It is left for future, possibly numerical, work to determine the Pr and Sc dependencies of the stability limit of fingers.

Another open issue concerns the role of the boundaries. Visualizations suggest that fingers are born as Rayleigh-Taylor instabilities in the chemical boundary layers. If the stabilizing temperature gradient is large enough, these grow into fingers, whereas if it is too small, they merge into a convection roll. Finger convection in the oceans frequently occurs in vertically stacked horizontal layers with layers of finger convection separated by layers mixed by ordinary convection. There are no solid walls limiting the fingers in this system and it remains to be seen whether fingers still exist for $|\Lambda|$ as small as $1/30$ in this case.

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